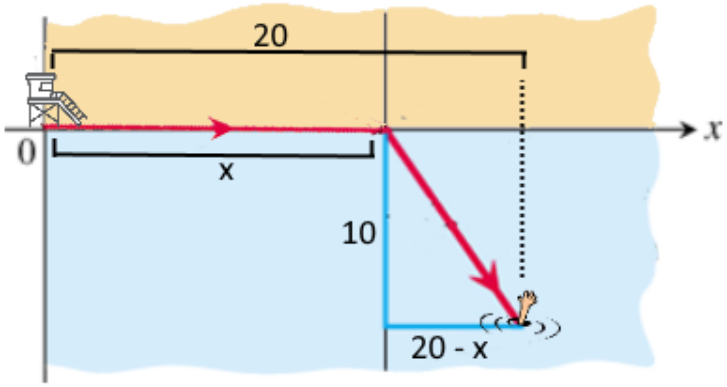
MAT 21A HW7

Due Tuesday September 7th

1. Consider the function
$$f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^2 + \frac{2}{3}x$$
over it's entire domain of all real numbers.
   (a) Find all the critical points of f.
   (b) Find all the inflection points of f (where $f''(x) = 0$)
   (c) Find all the intervals where f is concave, and find all the intervals where f is convex.
   (d) Find and classify all the local extrema of f.
   (e) Find the global minimum of f, and explain why f doesn't have a global maximum.

2. Evaluate the following limits (Hint: Use L'hopitals Rule)
   (a) $\lim_{x\to 0} \dfrac{\tan(x)}{x}$
   (b) $\lim_{x\to 2} \dfrac{2^x - x^2}{2 - x}$
   (c) $\lim_{x\to 0} \dfrac{\cos(x) - 1 + \frac{1}{2}x^2}{x^4}$
   (d) $\lim_{x\to 0^+} \sin(x)\ln(x)$
   (e) $\lim_{x\to \infty} \dfrac{\sqrt{x+5}}{x}$

3. You are a life guard on duty, and you are sitting along the shore. Suddenly, you spot a swimmer drowning $20m$ down the coast, and $10m$ out to sea. You can run at $6m/s$ through the sand and swim at $2m/s$ through the ocean. You quickly realize that the fastest route is to run along the water for some distance, and then jump in the water an swim directly towards the swimmer. At what point on the beach (how many meters down the coast) should you enter the water if you want to reach the swimmer as soon as possible?



4. A cola company wants to produce cylindrical cola cans out of aluminum. They are trying to minimize costs, so they want to minimize the surface area of the cans in order to save money on sheet metal.

   (a) If the cylindrical can has a radius of r and a height of h, what is the volume and surface area of the can? (For the surface area, don't forget to count the top and bottom of the can.)

   (b) The company wants to produce cans that can hold $350cm^3$ of soda. What is the minimum surface area such a can could have?

   (c) If aluminum sheet costs \$0.01 per $100cm^2$, how much does the aluminum in a single optomized can cost?

