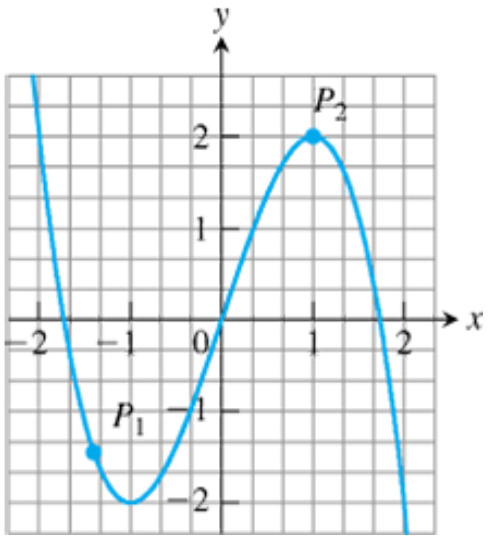
MAT 21A HW3

Due Friday August 20th

1. In the following picture, use the grid to estimate the slope of the tangent line at points P_1 and P_2.



2. Use the definition of a derivative to show that

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

3. The function $f(x) = \sqrt{|x|}$ is continuous at $x = 0$. Is it differentiable at $x = 0$? Explain your answer.

4. Differentiate the following functions:

   (a) $-3x^3 + 2x^2 - 5x + 1$

   (b) $x + x^2 \cdot 2^x + x^3 \cdot 3^x$

   (c) $3^{2x} \cdot 4^x$ (Hint: Try using exponent identities to rewrite this function)

   (d) $\dfrac{x-1}{x+1}$

   (e) $\dfrac{x^\pi}{e^x} - \dfrac{e^x}{x^\pi}$

   (f) $\dfrac{\left(\frac{1}{5}\right)^x + x}{x^2+1}$

5. Evaluate the following fourth derivative:

$$\frac{d^4}{dy^4}\ y \cdot e^y$$

6. This year skateboarding made its debut at the Olympics. Imagine you are an Olympic skateboarder. You need to jump at least 1 meter in the air in order to earn enough points to get a gold medal. When you jump off the top of the half pipe, you are going $5m/s$. To make the math easier, lets use an approximation for acceleration due to gravity at $-10m/s^2$.

   Your height above the rim of the half pipe in meters as a function of time in seconds is $h(t) = \frac{1}{2}at^2 + v_0t$ where a is acceleration due to gravity and v_0 is initial velocity.

   (a) Find the function $v(t)$, which is your velocity in meters per second as a function of time in seconds.

   (b) What is your velocity when you are at the highest point in the air? Explain your answer.

   (c) At what point in time are you are the highest point in the air?

   (d) How high is your jump? Do you get a gold medal?

   (e) What is the minimum initial velocity you needed to be traveling at to make the 1 meter jump? (Hint: If you get an answer of $4.5m/s$, you have the wrong answer.)

7. You are an airline pilot coming in for a landing at an airport with a short, $400m$ runway. Your velocity is a bit high, and so you need to determine if it is safe to land, or if you have to abort the landing and go around for another attempt. Your plane will touch down $150m$ from the start of the runway, at $50m/s$. Applying the brakes will result in acceleration of $-4m/s^2$.

   (a) Come up with a function which would give the distance traveled past the start of the runway as a function of seconds after the plane touches down.

   (b) If you land the plane, will you be able to stop before the end of the runway?

   (c) Lets suppose your plane is equipped with emergency breaks that can apply a higher than normal decelerating force. What rate of deceleration would be required to stop the plane before the end of the runway?

   (d) You forgot the button for the emergency breaks, so you fly around and go in for another landing. What is the furthest you can touch down from the start of the runway in order to safely land the plane?