Final Exam

September 10th, 6:10pm - 7:50pm

You must take this test yourself. You may not collaborate with anybody else, or use any outside resources. Calculators are not allowed. At the start of the exam, download "Final.pdf" from the assignments tab of Canvas. You must write your solutions on blank pieces of paper, and stay in the Zoom room with your camera on until after you upload your solutions. If you have any questions during the exam, private message Hans in the Zoom chat. After you finish the exam, scan your solutions into a pdf and upload them to Gradescope. If for some reason you cannot upload your solutions to Gradescope, email them to hoberschelp@math.ucdavis.edu. Other than these previously mentioned uses, you cannot use the internet or a computer.

Remember to SHOW YOUR WORK. Unsupported answers will not receive full credit, and might receive zero credit. Good luck!

Question	Points
1	15
2	10
3	10
4	10
5	15
6	10
7	20
8	10
Total	100

1. The following limits exist. Find what they equal. Remember to show your work.

   (Hint: For some of these limits L'hopitals Rule might be useful. For others, the Sandwich Theorem may be useful)

   (a) $\lim_{\theta \to \frac{\pi}{4}} \dfrac{\sin(\theta) - \cos(\theta)}{\theta - \frac{\pi}{4}}$ (3 points)

   (b) $\lim_{x \to \infty} \dfrac{\sin(\frac{1}{x})}{x^2}$ (3 points)

   (c) $\lim_{z \to 0^+} \dfrac{\ln(z)}{\ln(\sin(z))}$ (3 points)

   (d) $\lim_{y \to \pi} \dfrac{\cos^2(y)}{y}$ (3 points)

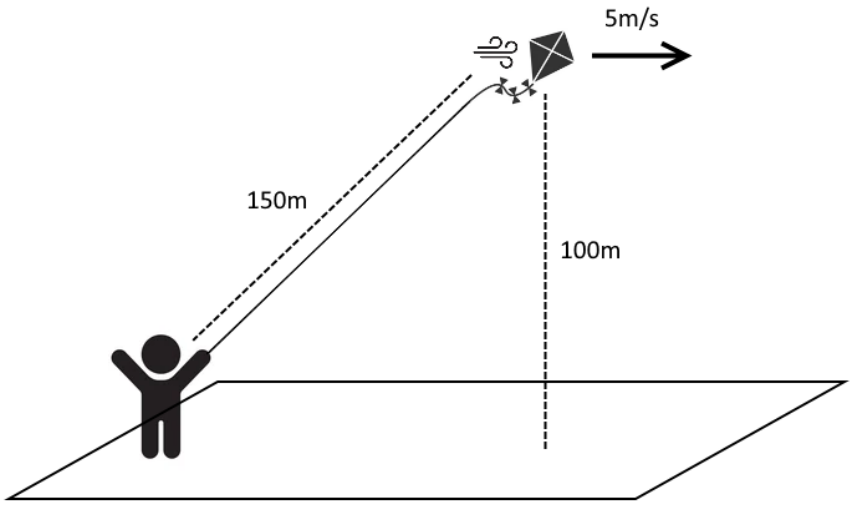
   (e) $\lim_{b \to 0} \dfrac{\cos(b^{-2})}{b^{-4}}$ (3 points)

2. Recall that our two equivalent definitions of differentiation are:

$$\frac{d}{dx}f(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \qquad \frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Using either one of these definitions, show that $\dfrac{d}{dx}\left(\dfrac{x+1}{x-1}\right) = \dfrac{-2}{(x-1)^2}$

3. A child is flying a kite. At this moment, the kite is 100 meters above the ground, and the kite is 150 meters away from the child. The wind is blowing the kite 5 meters per second horizontally away from the child. At what rate must the child let out string? (Equivalently, at what rate is the distance between the kite and the child increasing?)



4. Consider the function

$$g(x) = 2^{x+1} - 4^{x-1}$$

Find and classify all ***global*** extrema over the interval $[0, 3]$.

5. Consider the function $f(x) = \ln(x^2+3) - \frac{1}{2}x$ over its entire domain of all real numbers.

   (a) On what intervals is f increasing, and on what intervals is f decreasing? (6 points)

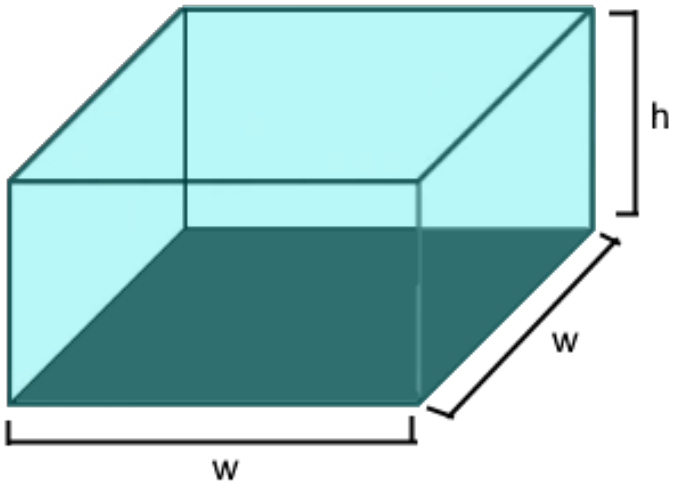
   (b) On what intervals is f convex, and on what intervals is f concave? (6 points)

   (c) Find and classify all ***local*** extrema of f. (3 points)

6. Prove that the equation $x^3 = \cos(x)$ has a solution.

   (Hint: Remember the Intermediate Value Theorem)

7. An aquarium is building a fish tank to house its new fish. The plans call for the tank to have a SQUARE base made of steel, and sides made of glass. The tank will have an open top. Steel costs \$50 per square meter, and glass costs \$20 per square meter. If the aquarium has a budget of \$600 for materials, what is the maximum volume the tank can have?



8. Determine whether the following statements are TRUE or FALSE. For this problem only, you do not need to show any work.

   (a) For a function f, the number of values of x at which $f(x)$ achieves a global maximum is either 0 or 1. (2 points)

   (b) If a function f is continuous over the interval $[a, b]$, then it is differentiable over the interval (a, b). (2 points)

   (c) If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minimum at c. (2 points)

   (d) If $f''(c) = 0$ then f does not have any local extrema at c. (2 points)

   (e) The limit $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x} \right)$ exists. (2 points)