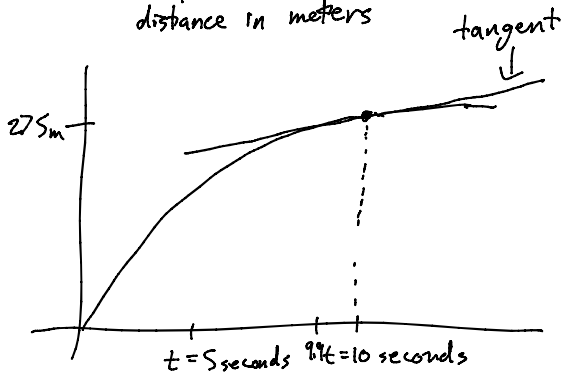
Wednesday, August 4, 2021 6:06 PM

Chapter 2.1

You are driving on a highway at $108^{km/hr}$ ($30^{m/s}$). You see a sign up ahead lowering the speed limit to $90^{km/hr}$ ($25^{m/s}$) and a cop waiting right next to the sign on the edge of the highway. You hit the breaks, and your distance traveled over time is given by $d(t) = 30t - \frac{1}{4}t^2$ and you pass the speed trap right at $t = 10$ seconds.

($d(t)$: distance in meters)



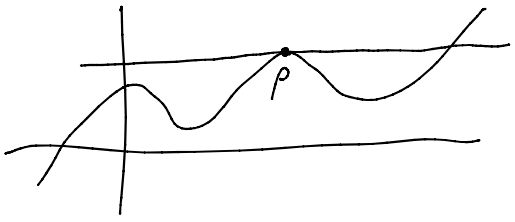
t	$d(t)$
0	0
5	143.75
9	249.75
9.9	272.4975
10	275

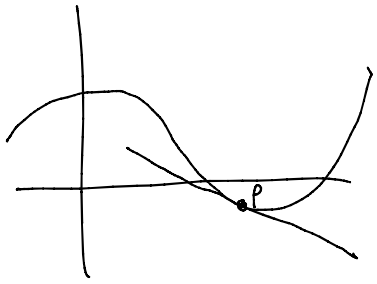
$$\text{average speed (over last 10 seconds)} = \frac{d_{final} - d_{initial}}{t_{final} - t_{initial}} = \frac{275 - 0}{10 - 0} = 27.5^{m/s}$$

$$\text{average speed (over last 5 seconds)} = \frac{275 - 143.75}{10 - 5} = \frac{131.25}{5} = 26.25^{m/s}$$

$$\text{average speed (over last second)} = \frac{275 - 272.4975}{10 - 9.9} = \frac{2.5025}{0.1} = 25.025^{m/s}$$

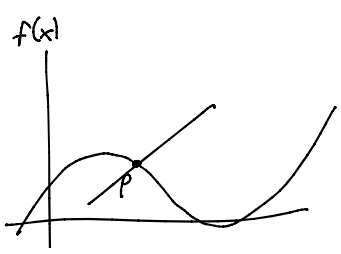
Def: A <u>tangent line</u> to a graph at a point p is the line that passes through p, but doesn't pass through any other points "near" p.

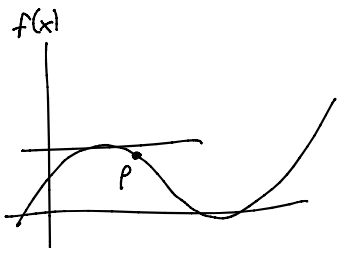


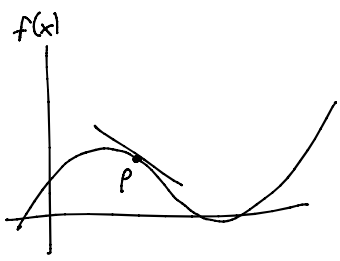


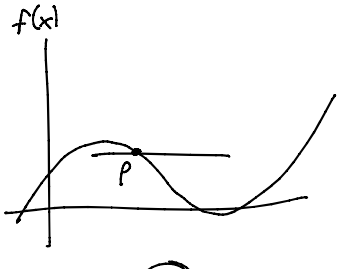
Def: The slope of a tangent line at p is called the <u>instantaneous rate of change</u> at p.

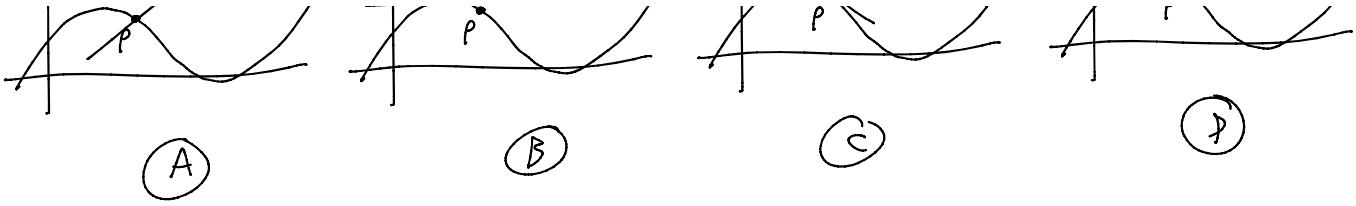
ex.





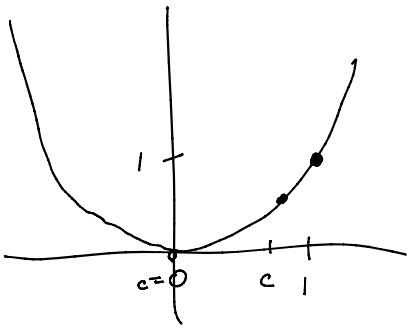






(A) (B) (C) (D)

Lets look at a more general example. $f(x) = x^2$. Lets try to figure out the instantaneous rate of change at $x=1$ (i.e. the point $(1,1)$)

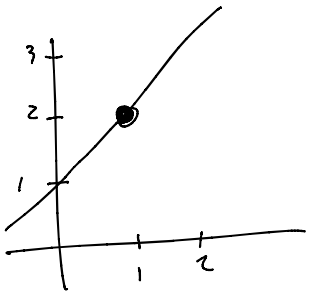


domain is all real numbers except for $c=1$

$\frac{1-1^2}{1-1} = \frac{0}{0}$ ✗

$$\text{slope} = \frac{f(1)-f(c)}{1-c} = \frac{1^2-c^2}{1-c} = \frac{1-c^2}{1-c} = \frac{(1+c)(1-c)}{1-c} = 1+c$$

$s(c)$ = slope when we use c to approx. rate of change



$s(1) = 1+1 = 2$

Monday, August 16, 2021 7:33 PM

3.4

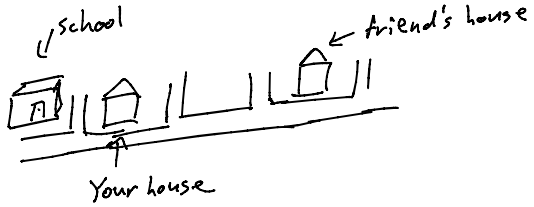
Recall our example from August 4th about the speed trap.

In that example, the ~~distance~~ displacement we traveled as a function of time was

$d(t) = 30t - \frac{1}{4}t^2$.

Def: The 1D (signed, positive or negative) distance from some point is called **displacement.**

ex.



displacement of your friend's house from your house is 2 block

dsplacement of the school from your house is -1 block.

Def: The rate of change of displacement is **velocity.**

$\frac{d}{dt}(30t) = 30 \cdot \frac{d}{dt}(t) = 30 \cdot 1$

$\frac{d}{dt}(-\frac{1}{4}t^2) = -\frac{1}{4}\frac{d}{dt}(t^2) = -\frac{1}{4} \cdot 2t = -\frac{1}{2}t$

ex. In our speed trap problem, the velocity is

$v(t) = d'(t) = 30 - \frac{1}{2}t$

Remember, our goal was to be at or under $25\ m/s$ at time $t = 10$ seconds.

$v(10) = 30 - \frac{1}{2} \cdot 10 = 25$ ✓ Yes, we did accomplish this! (11?)

By the way, how hard were we hitting our brakes.

Def: The rate of change of velocity is **acceleration.**

$\frac{d}{dt}(30) = 0$ $\frac{d}{dt}(-\frac{1}{2}t) = -\frac{1}{2}\frac{d}{dt}(t) = -\frac{1}{2}$

ex. In the same problem

$a(t) = v'(t) = -\frac{1}{2}\ m/s/s$

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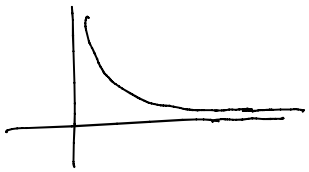
Ch. 2.6

Up the this point, we have talked about limits when x gets "closer and closer" to a real number.
But what about, x gets "greater and greater" or "less and less."

ex. What happens to $\frac{1}{x}$ as x gets "greater and greater"

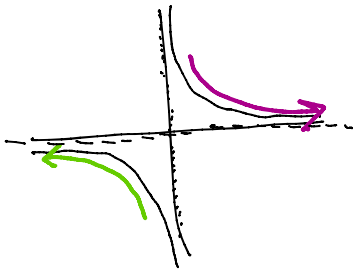
i.e. $x = 10$, $x = 100$, $x = 1{,}000{,}000$.

$\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1{,}000{,}000}$



Informal Def: If $f(x)$ gets closer to L as x increases, we say $\lim_{x\to\infty} f(x) = L$.
If $f(x)$ gets closer to L as x decreases, we say $\lim_{x\to-\infty} f(x) = L$.

ex. $f(x) = \frac{1}{x}$



$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$

NOT REQUIRED TO KNOW

Def: If, for all $\varepsilon > 0$ there exists M such that if $x > M$ then $|f(x) - \varepsilon| < L$, we say $\lim_{x\to\infty} f(x) = L$

If, for all $\varepsilon > 0$ there exists M such that if $x < M$ then $|f(x) - \varepsilon| < L$ we say $\lim_{x\to-\infty} f(x) = L$.

Thm: $\lim_{x\to\infty} \frac{1}{x^n} = 0$ for any real number $n > 0$

$\lim_{x\to\infty} a^x = 0$ for any $0 < a < 1$

$\lim_{x\to\infty} k = k$ for any constant k

$\lim_{x\to-\infty} \frac{1}{x^n} = 0$ for any real number $n > 0$

$\lim_{x\to-\infty} a^x = 0$ for any $1 < a$

$\lim_{x\to-\infty} k = k$ for any constant k.

Thm: The following limit laws also apply for limits to ∞ or $-\infty$.

$\lim$ $f(x) = M$ then

Thm: The following limit laws also apply for limits to ∞ or $-\infty$.

If f and g are functions and $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} g(x) = M$, then

1) $\lim_{x\to\infty} (f(x)+g(x)) = L+M$
2) $\lim_{x\to\infty} (f(x)\cdot g(x)) = L\cdot M$
3) $\lim_{x\to\infty} (f(x)-g(x)) = L-M$
4) $\lim_{x\to\infty} \left(\frac{f(x)}{g(x)}\right) = \frac{L}{M}$ if $M \neq 0$.

ex. $\lim_{x\to\infty} \frac{(\frac{1}{2})^x}{2+\frac{1}{x}} = \frac{\lim_{x\to\infty}(\frac{1}{2})^x}{\lim_{x\to\infty}(2)+\lim_{x\to\infty}(\frac{1}{x})} = \frac{0}{2+0} = 0$

ex. $\lim_{x\to\infty} \frac{x^2+1}{x^3+2x^2+x+3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^3}}{1+\frac{2}{x}+\frac{1}{x^2}+\frac{3}{x^3}}$

$= \frac{\lim_{x\to\infty}(\frac{1}{x}+\frac{1}{x^3})}{\lim_{x\to\infty}(1+\frac{2}{x}+\frac{1}{x^2}+\frac{3}{x^3})} = \frac{\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}\frac{1}{x^3}}{\lim_{x\to\infty}1+\lim_{x\to\infty}\frac{2}{x}+\lim_{x\to\infty}\frac{1}{x^2}+\lim_{x\to\infty}\frac{3}{x^3}} = \frac{0+0}{1+0+0+0} = 0.$

$\lim_{x\to\infty} \frac{x^2+1}{x^3+2x^2+x+3} = \lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to\infty} \frac{1}{x} = 0.$ ← only works for $\lim_{x\to\infty}$ polynomial/polynomial

ex. $\lim_{x\to-\infty} \frac{-x^3+2}{5x^3+x+1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to-\infty} \frac{-1+\frac{2}{x^3}}{5+\frac{1}{x^2}+\frac{1}{x^3}} = \frac{-1+0}{5+0+0} = -\frac{1}{5}$

$\lim_{x\to-\infty} \frac{-x^3}{5x^3} = \lim_{x\to-\infty} -\frac{1}{5} = -\frac{1}{5}$

Thm: Sandwich Theorem

If $g(x) \le f(x) \le h(x)$ for all $x > M$ (for some M) and $\lim_{x\to\infty} g(x) = L$ and $\lim_{x\to\infty} h(x) = L$, then $\lim_{x\to\infty} f(x) = L$.

If $g(x) \le f(x) \le h(x)$ for all $x < M$ (for some M) and $\lim_{x\to-\infty} g(x) = L$ and $\lim_{x\to-\infty} h(x) = L$, then $\lim_{x\to-\infty} f(x) = L$.

ex. $\lim_{x\to\infty} \frac{\sin(x)}{x}$

$-1 \le \sin(x) \le 1$

$-\frac{1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$ for $x > 0$.

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} -\frac{1}{x} = -\lim_{x\to\infty} \frac{1}{x} = -0 = 0.$

So $\lim_{x\to\infty} \frac{\sin(x)}{x} = 0.$

ex. $\lim_{x\to-\infty} \frac{5+3\cos(x)+x}{x^2}$

$2 \le 5+3\cos(x) \le 8$

$\frac{2}{x^2} \le \frac{5+3\cos(x)}{x^2} \le \frac{8}{x^2}$ for all $x < 0$.

ex. $\lim_{x\to-\infty} \frac{5+3\cos(x)+x}{x^2}$

$= \lim_{x\to-\infty} \frac{5+3\cos(x)}{x^2} + \underbrace{\lim_{x\to-\infty} \frac{x}{x^2}}_{\lim_{x\to-\infty}\frac{1}{x}=0}$

$= 0 + 0$

$= \boxed{0}$

$2 \le 5+3\cos(x) \le 8$

$\frac{2}{x^2} \le \frac{5+3\cos(x)}{x^2} \le \frac{8}{x^2}$ for all $x<0$.

$\lim_{x\to-\infty} \frac{2}{x^2} = 2\lim_{x\to-\infty}\frac{1}{x^2} = 2\cdot 0 = 0$

$\lim_{x\to-\infty} \frac{8}{x^2} = 0$.

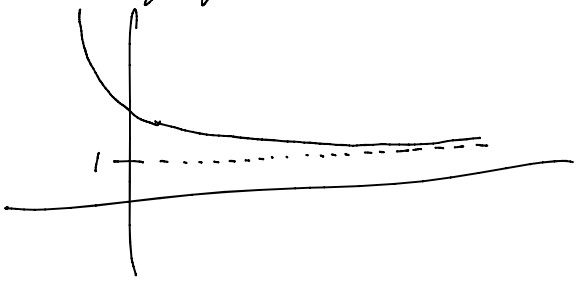
So $\lim_{x\to-\infty} \frac{5+3\cos(x)}{x^2} = 0$.

★ By the way, there is also a one-sided sandwich thm.

If $g(x) \le f(x) \le h(x)$ for $c<x<a$ and $\lim_{x\to c^+} g(x) = L$ and $\lim_{x\to c^-} h(x) = L$,

then $\lim_{x\to c^+} f(x) = L$.

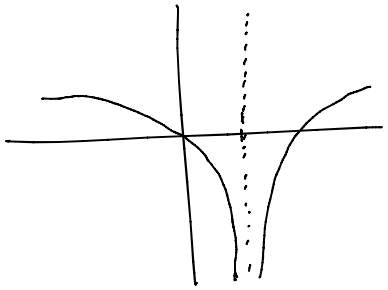
Consider the graph of $1+(\frac{1}{2})^x$



$\lim_{x\to\infty}(1+(\frac{1}{2})^x) = 1+0 = \boxed{1}$

Graphically, when $\lim_{x\to\infty} f(x) = A$ or $\lim_{x\to-\infty} f(x) = A$ we say f has a <u>horizontal asymptote</u> at A.

We can also have <u>vertical asymptotes.</u>



← This has a <u>vertical asymptote</u> at 1.

$\ln(|x-1|)$

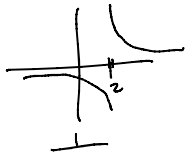
Note: $\lim_{x\to1^+} \ln(|x-1|)$ does not exist. But, to describe its behavior we write

$\lim_{x\to1^+} \ln(|x-1|) = -\infty$ ← it doesn't really "equal" $-\infty$

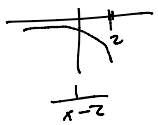
$\lim_{x\to1^-} \ln(|x-1|) = -\infty$

→ $\lim_{x\to1} \ln(|x-1|) = -\infty$

Q: Does $\lim_{x\to1} \ln(|x-1|)$ exist? <u>No</u>



Q: Does $\lim_{x\to 1} \ln(|x-1|)$ exist? No



ex. $\lim_{x\to 2^+} \frac{3}{x^2-3x+2} = \lim_{x\to 2^+} \frac{3}{(x-1)(x-2)} = \lim_{x\to 2^+} \left(\frac{3}{x-1}\cdot\frac{1}{x-2}\right) = \infty$

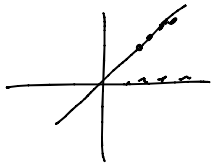
positive for 2.1, 2.2, 3, -- all $x>2$; positive when $x>2$

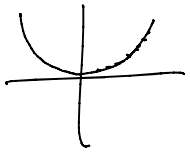
ex. $\lim_{x\to -1^-} \frac{(x+1)x}{(x+1)^{3/2}} = \lim_{x\to -1^-} \frac{x}{(x+1)^2}$ ← close to -1, more importantly negative for any $x<-1$ ← (anything)^2 = positive.

As we approach -1 from the left $\frac{x}{(x+1)^2}$ will always be $\frac{\text{negative}}{\text{positive}}$ = negative,

so $\lim_{x\to -1^-} \frac{(x+1)x}{(x+1)^3} = -\infty$

ex. $\lim_{x\to\infty} x = \infty$ $\lim_{x\to\infty} x^2 = \infty$.





ex. $\lim_{x\to -\infty} \frac{x^3+x^2+1}{2x^2+7}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to -\infty} \frac{x+1+\frac{1}{x^2}}{2+\frac{7}{x^2}} = -\infty$

For very negative x, $x+1+\frac{1}{x^2} < 0$.

$2+\frac{7}{x^2} > 0$.

$\frac{\text{positive}}{\text{negative}}$ = negative.

Thm: If $f(x)$ and $g(x)$ are polynomials and $f(x)$ has a higher degree than $g(x)$. Then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$ or $\lim_{x\to\infty} \frac{f(x)}{g(x)} = -\infty$

and $\lim_{x\to -\infty} \frac{f(x)}{g(x)} = \infty$ or $\lim_{x\to -\infty} \frac{f(x)}{g(x)} = -\infty$.

* x^3+3x+7 has degree 3

y^2+4 has degree 2

in general, the degree is the value of the highest power. i.e. $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has degree n.

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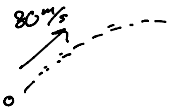
Ch. 3.4 continued

Physics Thm: An object moving under constant acceleration has displacement as a function of time given by

$$d(t) = d_0 + v_0 t + \frac{1}{2} a t^2$$

where d_0 is the displacement at time $t=0$ (initial displacement)
and v_0 is the velocity at time $t=0$ (initial velocity)
and a is the acceleration.

ex. A professional golf can hit a golf ball at $80^m/_s$.



Lets say a golfer, (aiming for a high angle) imparts a rate of change in the height of the ball above the ground of $60^m/_s$.

How long until the ball hits the ground? Assume gravitational acceleration is $10^m/_{s^2}$.

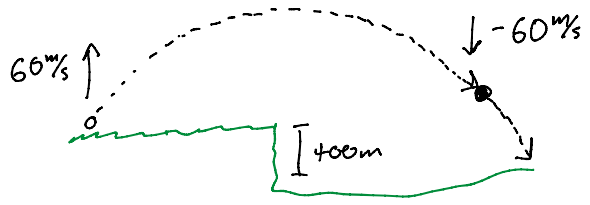
$$h(t) = h_0 + v_0 t + \frac{1}{2} a t^2.$$
$$= 0 + 60 \cdot t + \frac{1}{2} \cdot (-10) t^2 = 60t - 5t^2$$

$$60t - 5t^2 = 0$$
$$60 - 5t = 0$$
$$60 = 5t$$
$$t = 12 \text{ seconds}$$

What is the velocity (vertically) that the ball hits the ground at?

$$v(t) = h'(t) = \frac{d}{dt}(60t - 5t^2) = 60\frac{d}{dt}t - 5\frac{d}{dt}t^2 = 60 - 5 \cdot 2t = 60 - 10t$$

$$v(12) = 60 - 10 \cdot 12 = 60 - 120 = -60^m/_s.$$



Now lets imagine this golfer is on the edge of a 400m cliff.
How long until the ball hits the ground?
What is the velocity at that time?

$$400 + 60t - \frac{1}{2} \cdot 10 t^2 = 400 + 60t - 5t^2$$

144 + 320 = 464

What is the velocity at that time?

$h(t) = 400 + 60t - \frac{1}{2}\cdot 10t^2 = 400 + 60t - 5t^2$

$144 + 320 = 464$

$-5t^2 + 60t + 400 = 0$

$t^2 - 12t - 80 = 0$

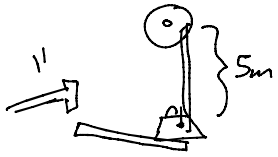
$t = \frac{12 \pm \sqrt{12^2 + 4\cdot 80}}{2}$

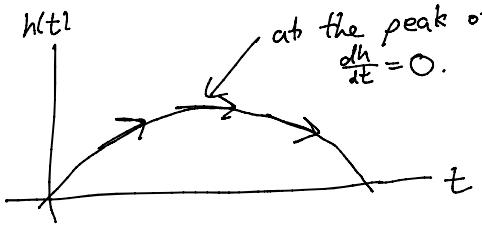
$= 6 \pm \sqrt{116} \approx 16.8$ seconds

$v(t) = h'(t) = \frac{d}{dt}(400 + 60t - 5t^2) = \cancel{\frac{d}{dt}(400)} + 60\frac{d}{dt}(t) - 5\cdot\frac{d}{dt}(t^2) = 60 - 10t$

$v(16.8) = 60 - 10\cdot 16.8 = 60 - 168 = -108$ m/s

ex. Imagine you are at a carnival, playing one of those "test your strength" games. The bell is 5m in the air. Gravity pulls at $10 m/s^2$. What velocity do you need to launch the weight at in order to hit the bell? (is the minimum)





$h(t) = h_0 + v_0 t - \frac{1}{2}at^2 = 0 + v_0 t - \frac{1}{2}\cdot 10t^2 = v_0 t - 5t^2$

$v(t) = h'(t) = \frac{d}{dt}(v_0 t) - \frac{d}{dt}(5t^2) = v_0 - 10t$

$v_0 - 10t = 0$

$v_0 = 10t$

$t = \frac{v_0}{10}$

$h\left(\frac{v_0}{10}\right) = 5$

$v_0\cdot\frac{v_0}{10} - 5\cdot\left(\frac{v_0}{10}\right)^2 = 5$

$\frac{v_0^2}{10} - 5\cdot\frac{v_0^2}{100} = 5$

$\frac{v_0^2}{10} - \frac{v_0^2}{20} = 5$

$\frac{2v_0^2}{20} - \frac{v_0^2}{20} = 5$

$\frac{2v_0^2 - v_0^2}{20} = 5$

$\frac{v_0^2}{20} = 5$

$v_0^2 = 100$

$v_0 = 10$ m/s

Wednesday, August 18, 2021 6:52 PM

Ch. 3.5

Goal: Show $\frac{d}{dx}\sin(x) = \cos(x)$.

$$\frac{d}{dx}\sin(x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)+\sin(h)\cos(x)-\sin(x)}{h}$$

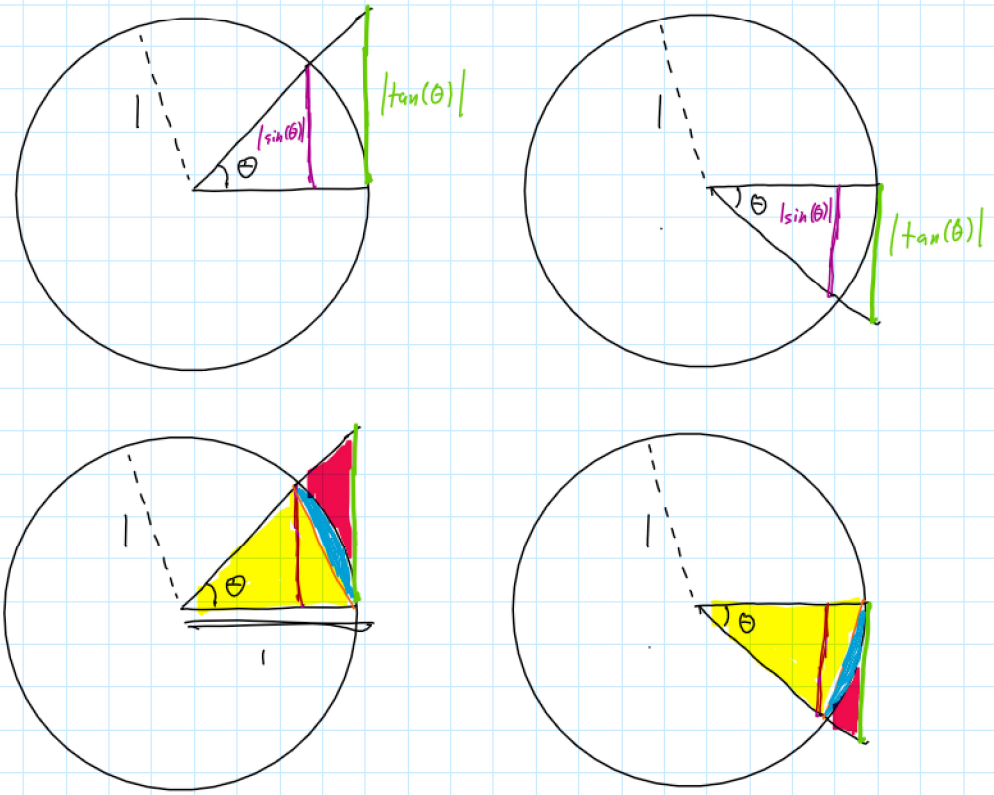
$$= \lim_{h\to 0}\frac{\sin(x)(\cos(h)-1)+\cos(x)\cdot\sin(h)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)(\cos(h)-1)}{h} + \lim_{h\to 0}\frac{\cos(x)\cdot\sin(h)}{h}$$

$$= \sin(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} + \cos(x)\cdot\lim_{h\to 0}\frac{\sin(h)}{h}$$

* $\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$

Lets start by solving $\lim_{h\to 0}\frac{\sin(h)}{h}$.



$$\text{Area}(\text{yellow}) = \frac{1\cdot|\sin(\theta)|}{2} = \frac{|\sin(\theta)|}{2}$$

$$\text{Area}(\text{yellow}+\text{blue}) = \pi r^2\cdot\frac{|\theta|}{2\pi} = \frac{r^2\cdot|\theta|}{2} = \frac{|\theta|}{2}$$

(all assuming $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$)

$$\text{Area}(\text{yellow}+\text{blue}+\text{red}) = \frac{1\cdot|\tan(\theta)|}{2} = \frac{|\tan(\theta)|}{2}$$

Area() $= \pi r^2 \cdot \frac{|\theta|}{2\pi} = \frac{r^2 \cdot |\theta|}{2} = \frac{|\theta|}{2}$ $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Area() $= \frac{1 \cdot |\tan(\theta)|}{2} = \frac{|\tan(\theta)|}{2}$

$$\frac{|\sin(\theta)|}{2} \le \frac{|\theta|}{2} \le \frac{|\tan(\theta)|}{2}$$

$$\frac{|\sin(\theta)|}{|\sin(\theta)|} \le \frac{|\theta|}{|\sin(\theta)|} \le \frac{|\tan(\theta)|}{|\sin(\theta)|} \qquad \frac{\tan(\theta)}{\sin(\theta)} = \frac{\frac{\sin(\theta)}{\cos(\theta)}}{\sin(\theta)} = \frac{1}{\cos(\theta)}$$

$$1 \le \left|\frac{\theta}{\sin(\theta)}\right| \le \left|\frac{1}{\cos(\theta)}\right|$$

$$1 \ge \frac{\sin(\theta)}{\theta} \ge \cos(\theta) \quad \text{for } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \text{ and } \theta \neq 0$$

$\lim_{\theta \to 0} 1 = 1$ and $\lim_{\theta \to 0} \cos(\theta) = 1$. So $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$.

Now $\lim_{\theta \to 0} \frac{\cos(\theta)-1}{\theta} \cdot \frac{\cos(\theta)+1}{\cos(\theta)+1} = \lim_{\theta \to 0} \frac{\cos^2(\theta)-1}{\theta(\cos(\theta)+1)} = \lim_{\theta \to 0} \frac{-\sin^2(\theta)}{\theta(\cos(\theta)+1)}$

$= \underbrace{\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta}}_{=1} \cdot \underbrace{\lim_{\theta \to 0} \frac{-\sin(\theta)}{\cos(\theta)+1}}_{\frac{-\sin(0)}{\cos(0)+1} = \frac{-0}{2} = 0} = 1 \cdot 0 = 0$

$$\frac{d}{dx}\sin(x) = \sin(x) \underbrace{\lim_{h \to 0} \frac{\cos(h)-1}{h}}_{=0} + \cos(x) \cdot \underbrace{\lim_{h \to 0} \frac{\sin(h)}{h}}_{=1}$$

$$= \sin(x) \cdot 0 + \cos(x) \cdot 1 = \cos(x)$$

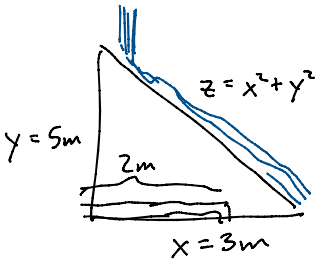
Homework: Show...

- $\frac{d}{dx}\cos(x) = -\sin(x)$
- $\frac{d}{dx}\tan(x) = \sec^2(x)$
- $\frac{d}{dx}\cot(x) = -\csc^2(x)$
- $\frac{d}{dx}\sec(x) = \sec(x) \cdot \tan(x)$
- $\frac{d}{dx}\csc(x) = -\csc(x) \cdot \cot(x)$

3.10

Monday, August 23, 2021 10:27 AM

ex.



$x^2 + y^2 = k$

$\frac{x}{y}$ remains constant.

$x = ky$

$3 = k \cdot 5$

$k = \frac{3}{5}$

$\frac{d}{dt}(x) = \frac{d}{dt}(ky)$

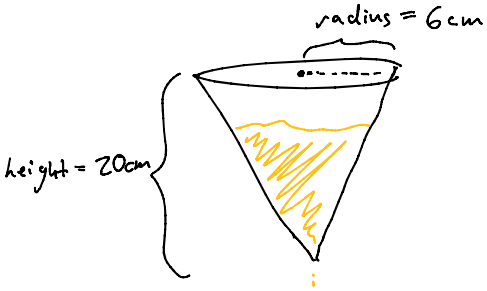
$\frac{dx}{dt} = k\frac{dy}{dt}$

If $\frac{dy}{dt} = 1mm/s = 0.001 m/s$.

$\frac{dx}{dt} = \frac{3}{5} \cdot 0.001 m/s = \frac{3}{5000} m/s$

ex. Sand is falling through an hour glass at a rate of $2 cm^3/s$.

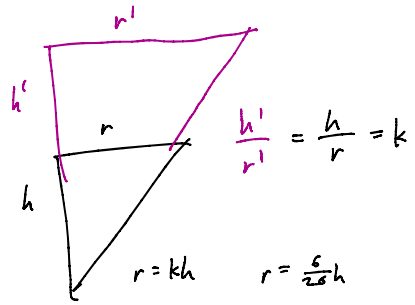
The top half of the hour glass is a cone with dimensions:



(a) find $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$. (b) Find $\frac{dh}{dt}$ in terms of $\frac{dh}{dt}$.

The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.

At what rate is the height of the sand changing over time?



$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r \cdot \frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$

$= \frac{1}{3}\pi\left(2 \cdot \left(\frac{6}{20}\right)^2 \frac{dh}{dt} h^2 + \left(\frac{6}{20}h\right)^2 \cdot \frac{dh}{dt}\right) = \pi\left(\frac{6}{20}\right)^2 h^2 \cdot \frac{dh}{dt}$

When the height of the sand is 10cm, what is $\frac{dh}{dt}$?

$2 = \pi \cdot \left(\frac{6}{20}\right)^2 \cdot 10^2 \cdot \frac{dh}{dt} = \pi \cdot 9 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2}{9\pi}$

ex. The kinetic energy of a moving object is $E = \frac{1}{2}mv^2$, where m is the mass and

ex. The kinetic energy of a moving object is $E=\frac{1}{2}mv^2$, where m is the mass and v is the velocity.

(a) If a 7 kg block is flying 5 m/s upwards, and experiencing gravity of $-10\ m/s^2$, what is $\frac{dE}{dt}$?

$$\frac{dE}{dt} = \frac{1}{2}m\,2v\cdot\frac{dv}{dt} = mva$$
$$= 7\cdot 5\cdot(-10) = -350\ J/s = -350\ W$$

(b) Now suppose a $\frac{1}{100}$ kg ice cube is floating upwards in a cup of tea at $\frac{1}{10}$ m/s. The buoyancy force is causing an additional $\frac{1}{20}\ m/s^2$ of upward acceleration. Also, the ice cube is melting, so its mass is changing at $-\frac{1}{1000}$ kg/s.

$$\frac{dE}{dt} = \frac{1}{2}\frac{dm}{dt}v^2 + mv\cdot\frac{dv}{dt}$$
$$= \frac{1}{2}\left(-\frac{1}{1000}\right)\cdot\left(\frac{1}{20}\right)^2 + \frac{1}{100}\cdot\frac{1}{10}\cdot\frac{1}{20}$$